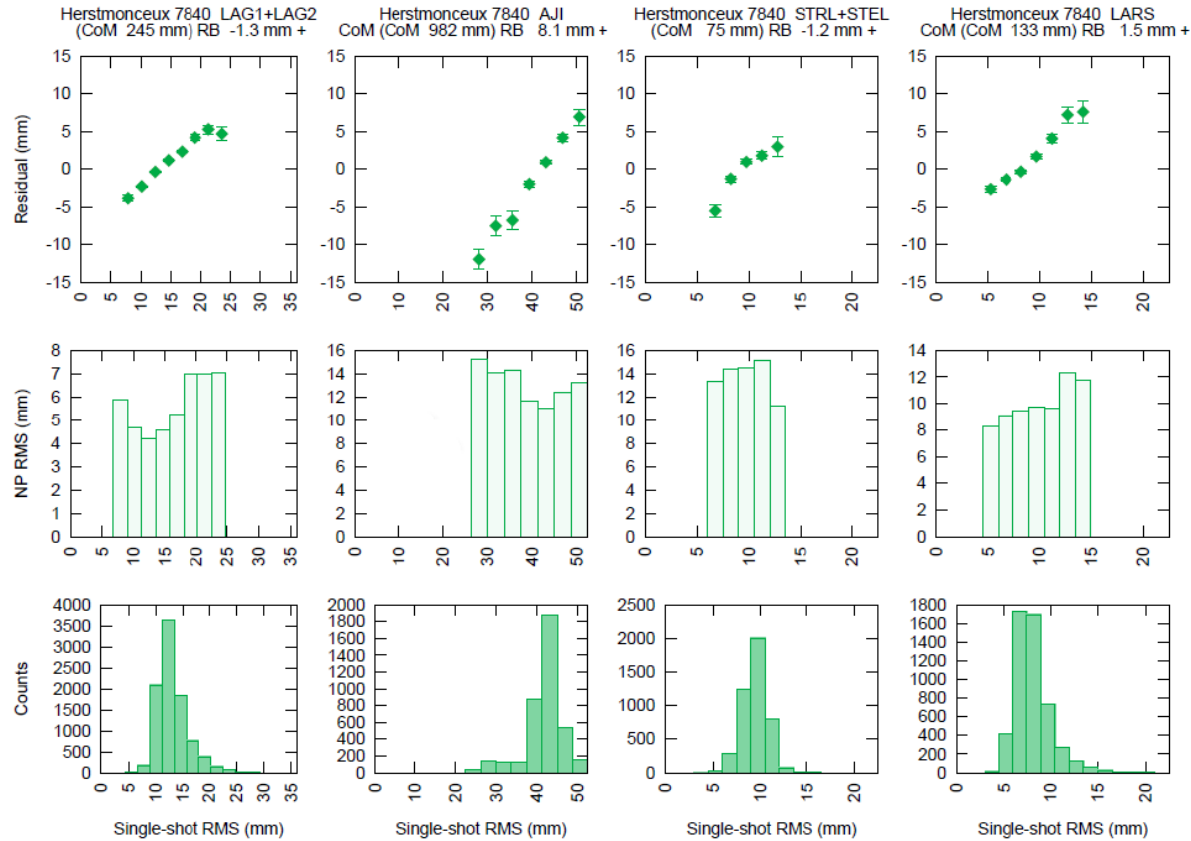# Variability of LAGEOS normal point sampling: causes and mitigation

José Rodríguez[1], Graham Appleby[1], Toshimichi Otsubo[2], Robert Sherwood[1], Matthew Wilkinson[1]
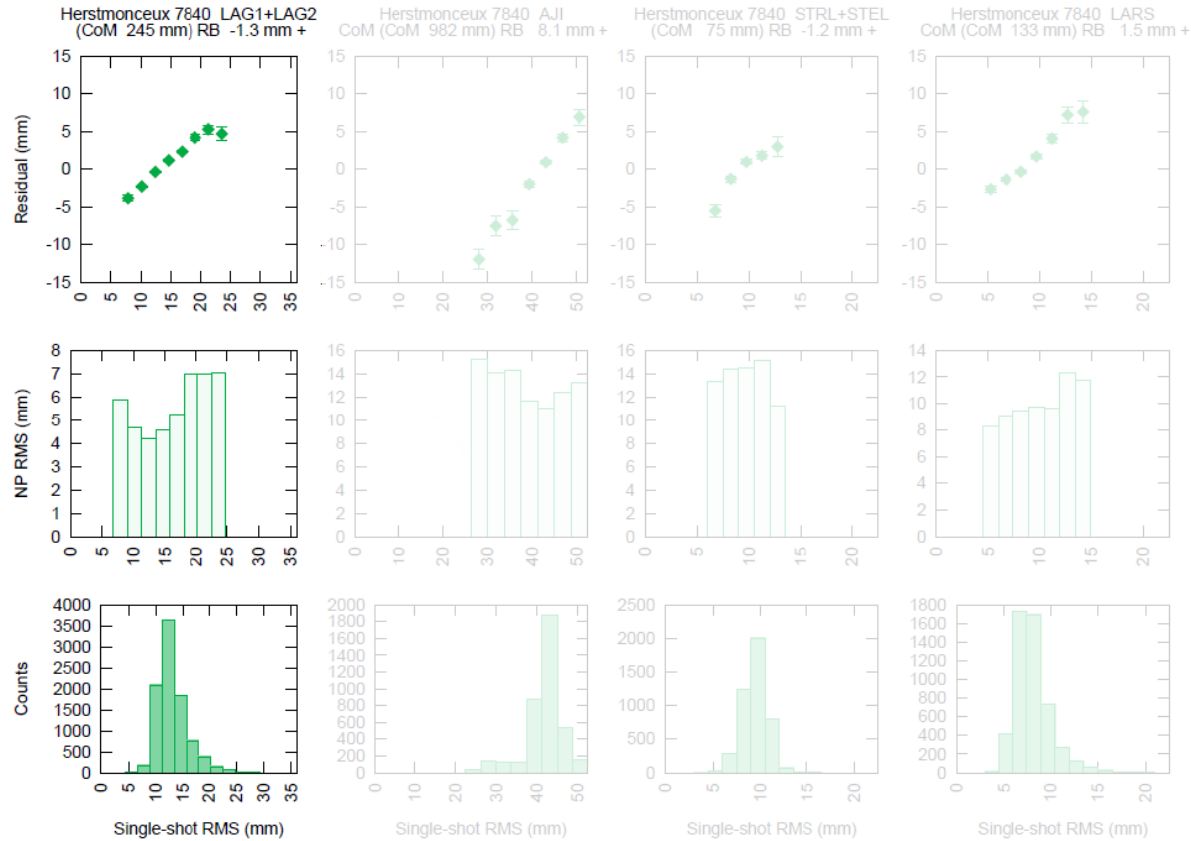
[1]BGS Space Geodesy Facility, UK
[2]Hitotsubashi University, Japan

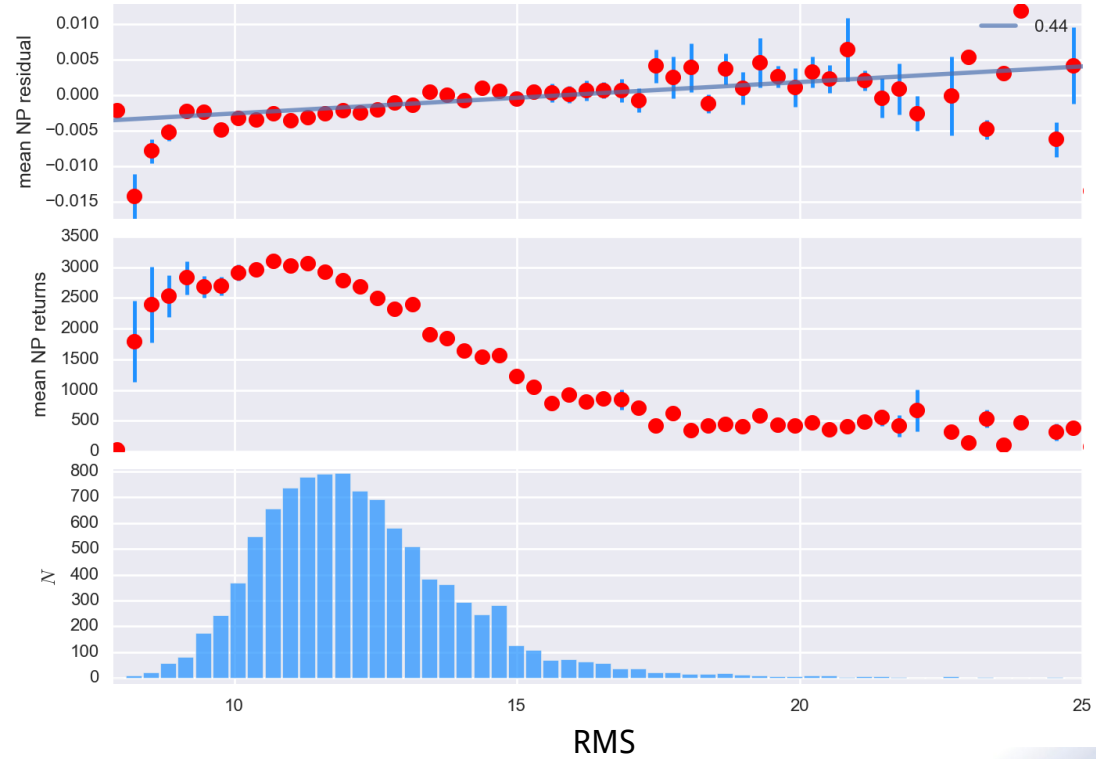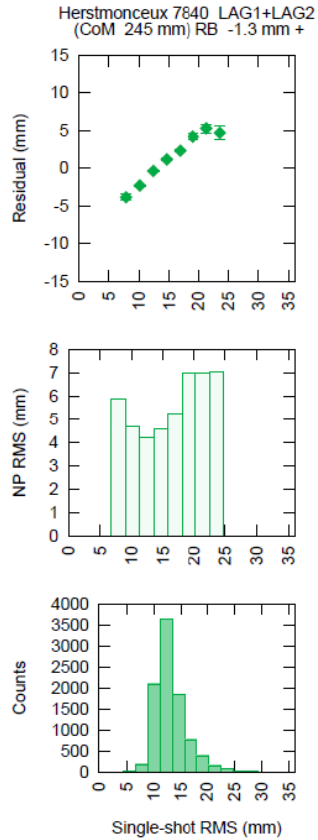# Hitotsubashi Univ. analysis: NP residual vs NP σ (c5++ POD, 2016.5 – 2017.4)

Hitotsubashi Univ. analysis: NP residual vs NP σ (c5++ POD, 2016.5 – 2017.4)

LAGEOS NP residual vs NP σ from SATAN orbital analysis, 2014.9 – 2017.5

7840 res vs s

Herstmonceux 7840 LAG1+LAG2
(CoM 245 mm) RB -1.3 mm +

Objectives

Consider some **causes** of NP **variability**

1. Physical sampling variability of retroreflector array

2. Variability from sampling the distributions of returns

3. Background noise

Examine **performance** of different **data reduction** schemes:
- simulated data
- empirical data

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1. Physical sampling variability

Task: Simulate laser pulses reflected off LAGEOS
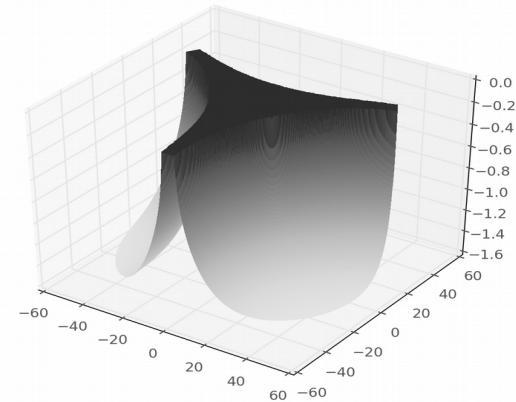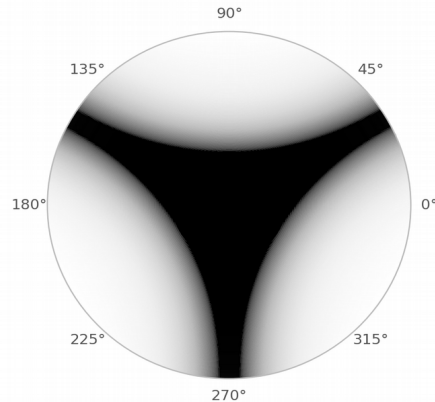
We need: Reflectivity map for a single retroreflector
              Array coordinates and clocking angles
              System noise
              Observation geometry

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We need: Reflectivity map for a single retroreflector
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Ray-tracing of a cube corner → reflectivity at all possible angles of incidence

Arnold D, Methods of calculating retroreflector-array transfer functions, 1978
Arnold D, Optical and IR transfer function of the LAGEOS retroreflector array, 1978
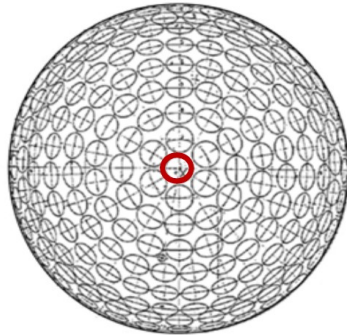
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Ray-tracing of a cube corner → reflectivity at all possible angles of incidence

For any given orientation, for all visible cube corners, compute incidence angle and retroreflector orientation →
       distances from origin, reflectivity x effective areas → array response
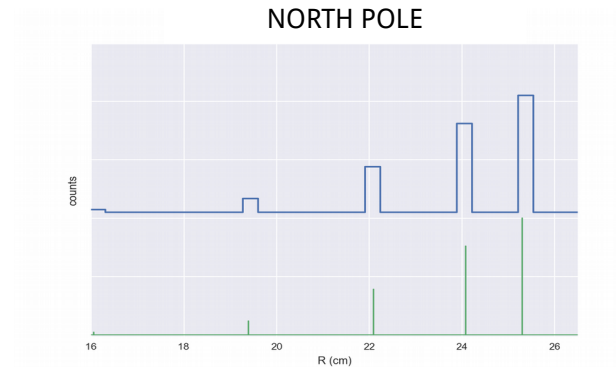
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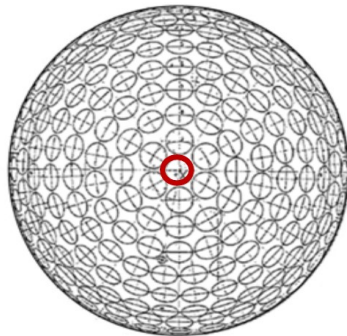NORTH POLE

1. Physical sampling variability

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NORTH POLE
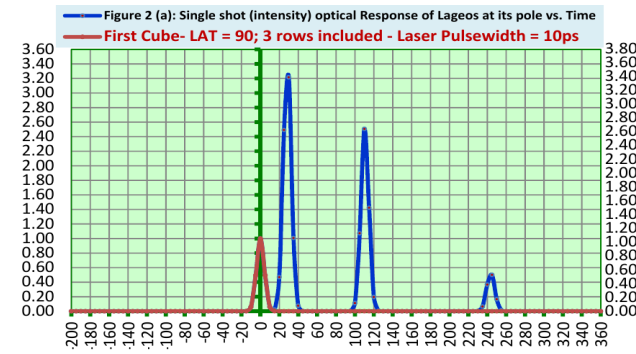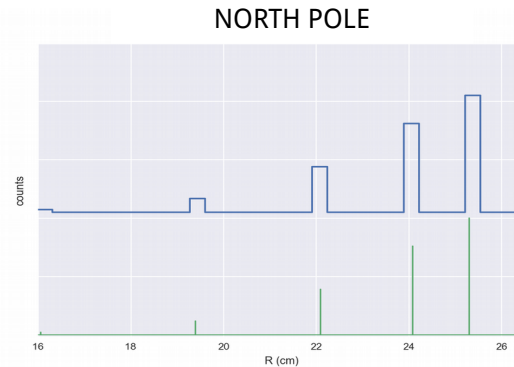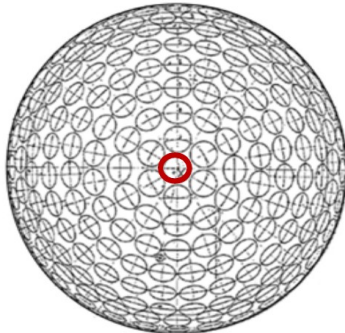
Varghese T, Zagwodzki T, Oldham T, Hu S. Attempt to further enhance ranging accuracy to LAGEOS through de-convolution of the target response. 18th IWLR 13-0417, Fujiyoshida, Japan, 2013.

REFLECTIONS OFF LAGEOS AT SOME RANDOM ORIENTATIONS
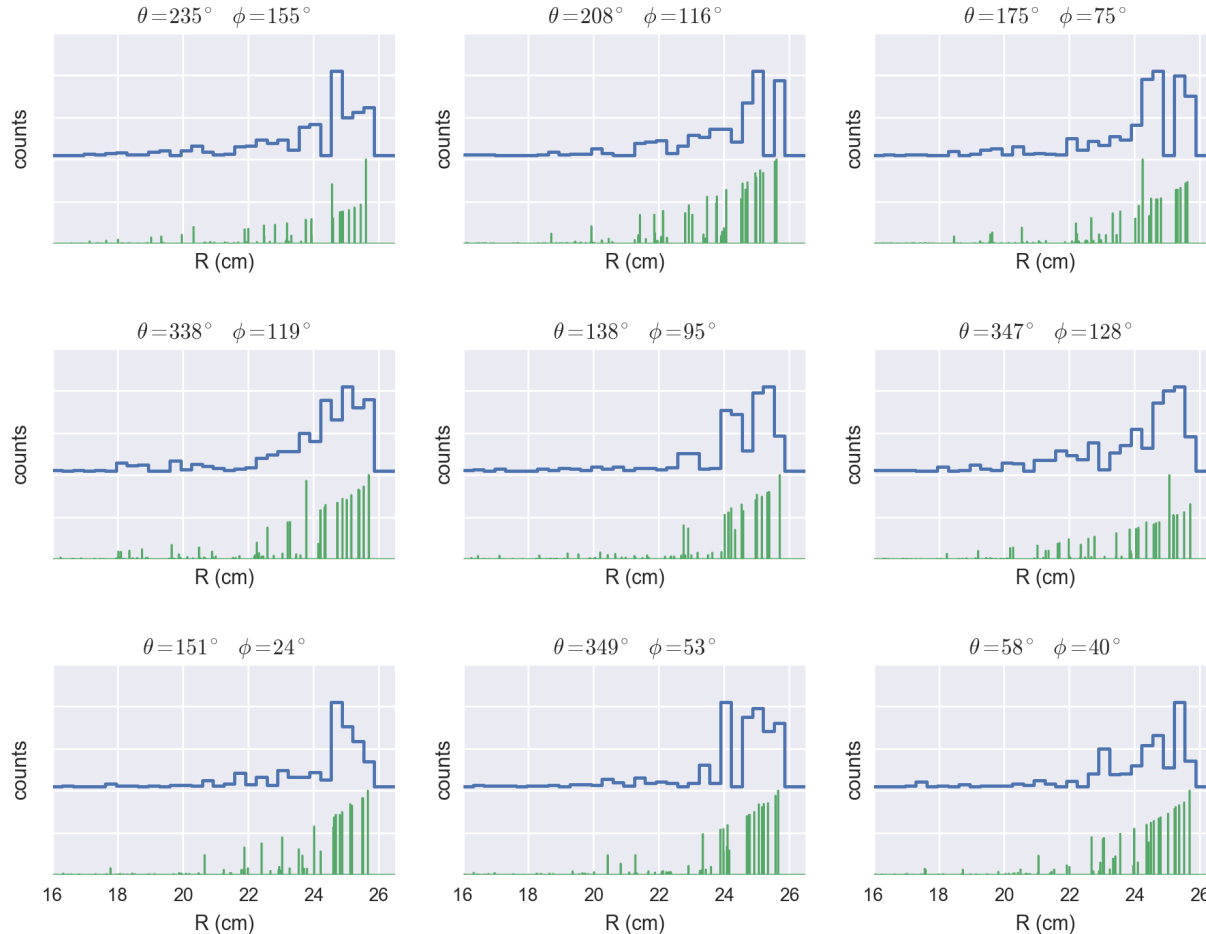
LAGEOS responses vary substantially:
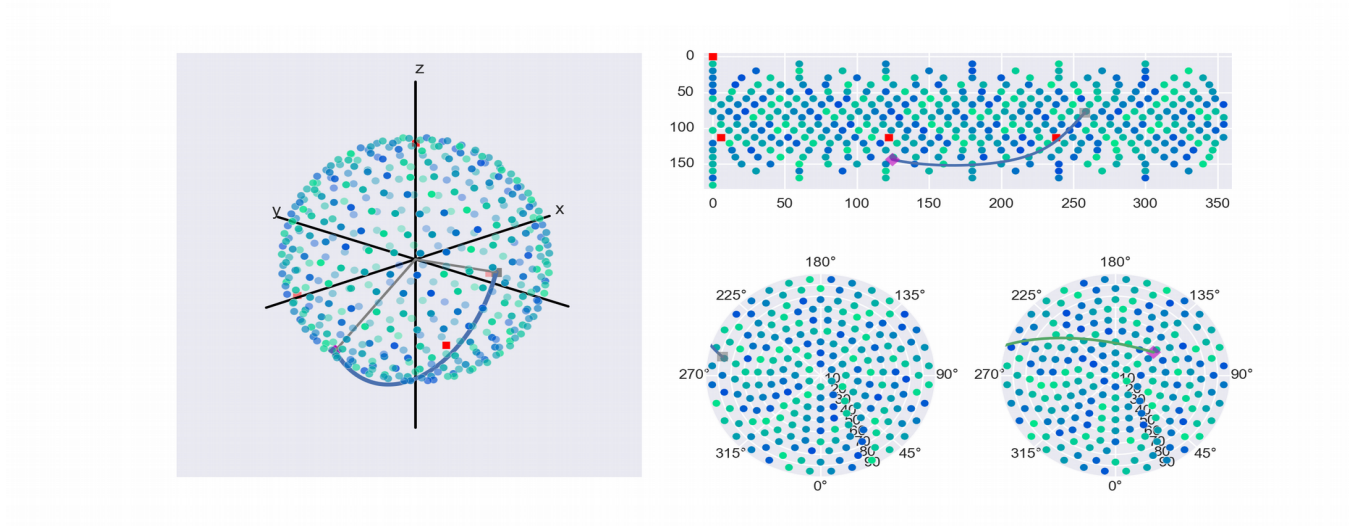
-response envelopes of different width and overall shape
-occasional multiple peaks
-late reflections at different positions and intensities

We can compute the response over a pass

LAGEOS surface sampled during a single pass

Assuming: an **arbitrary** LAGEOS orientation
                   insignificant satellite spin/tumble during ~1 hour

Compute instantaneous response for the orientations sampled over one pass

Incident beam orientation changes with satellite orbital motion and Earth rotation

We can compute the response over a pass
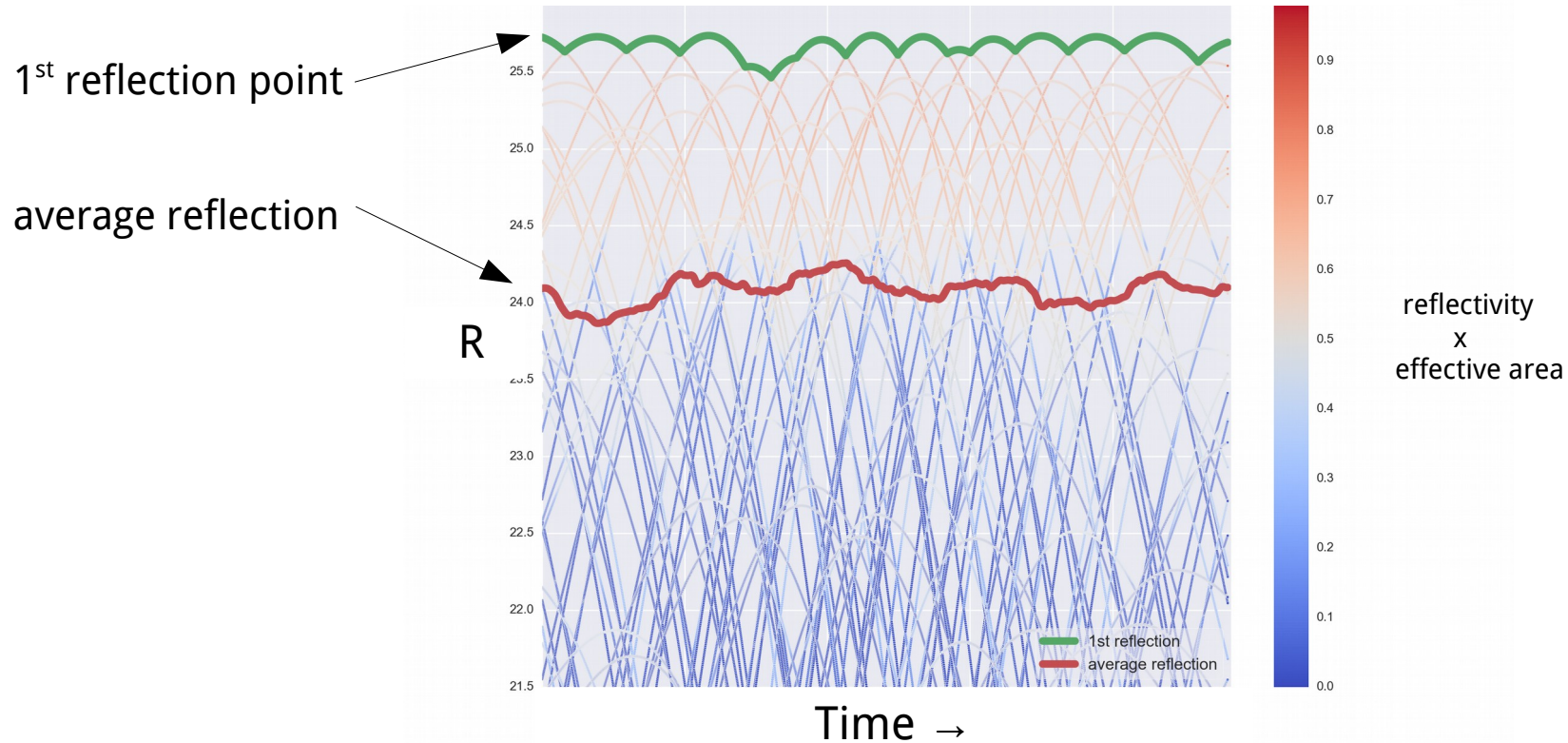
1st reflection point

average reflection

R

Time →

From the individual reflections we can compute the stability of different reflection points over a particular pass, e.g. first and average reflections

We can compute the response over a pass

1st reflection point
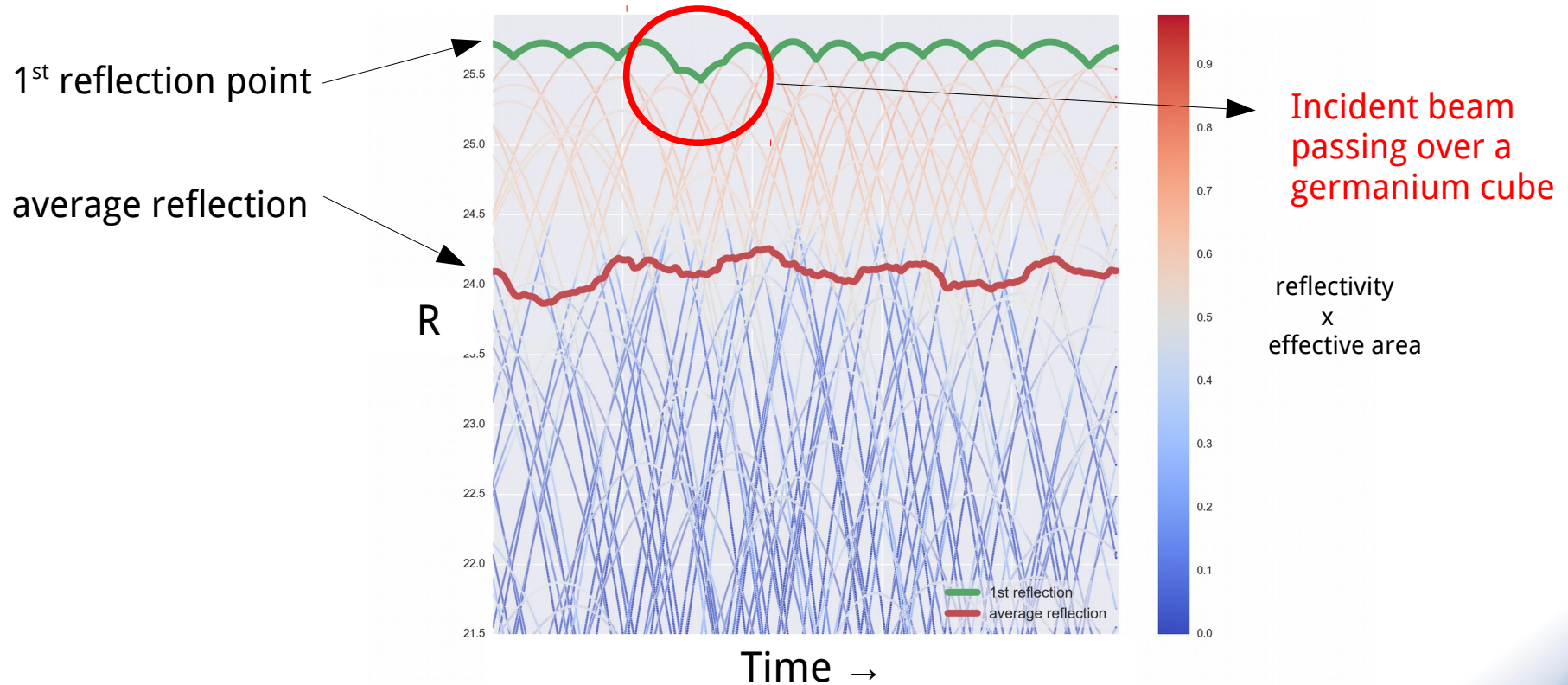
average reflection

Incident beam passing over a germanium cube

R

reflectivity
x
effective area

1st reflection
average reflection

Time →

From the individual reflections we can compute the stability of different reflection points over a particular pass, e.g. first and average reflections

...and compute the response over many passes

Assuming a **fixed**, arbitrary LAGEOS orientation:

      Compute responses for LAGEOS passes taken from one location over 6 months
      Form 2 minute normal points from simulated data
      Convolve NPs with system noise
      Reduce data with several methods

4773 simulated NPs: enough to examine the statistics of the reduction results

Reduction schemes tested:
      Herstmonceux scheme: 3σ iterative mean + Gaussian fit + $3\sigma_{Gauss}$ rejection

      N-iterative means
      Leading edge-based methods (Graz)

Pass reference point vs pass RMS

Reduction of simulated LAGEOS data (2017.0 - 2017.6)

Ref. point

RMS
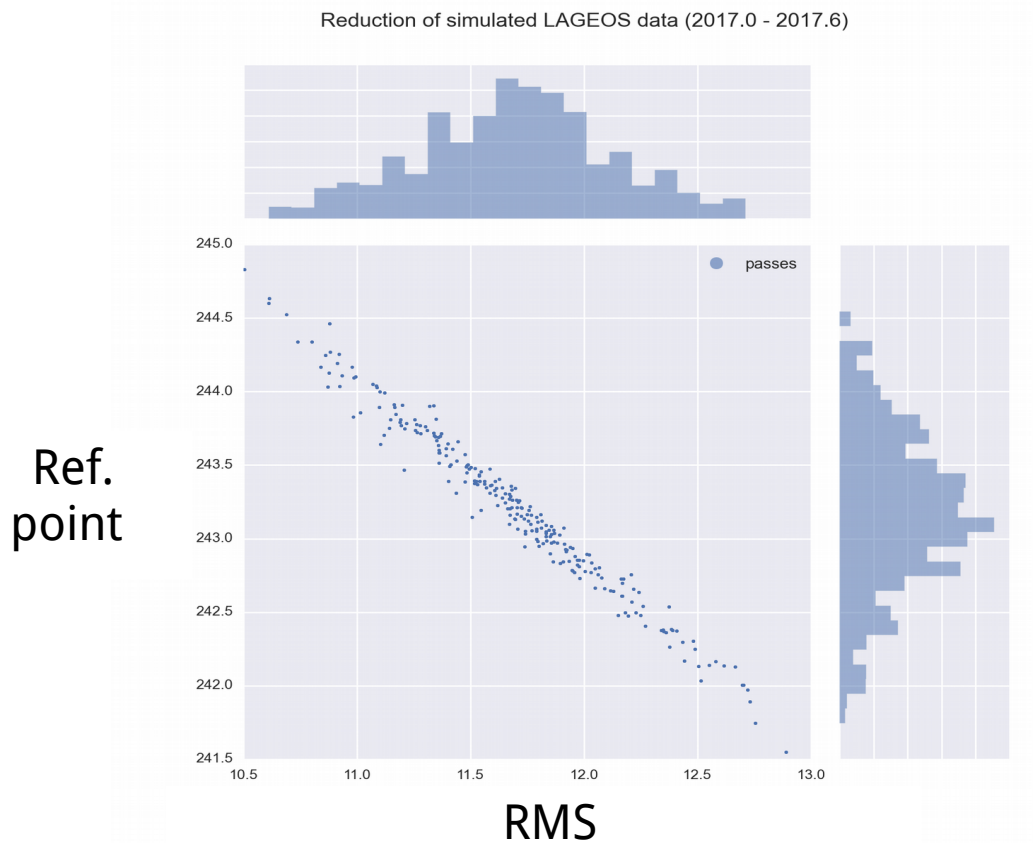
RMS peak-to-peak > 2 mm
Ref. Point peak-to-peak ~3 mm

(3 x Gauss reduction method)

Plotting, on a pass basis, the computed reference points against RMS shows a correlation between the two quantities

NP reference point vs NP RMS

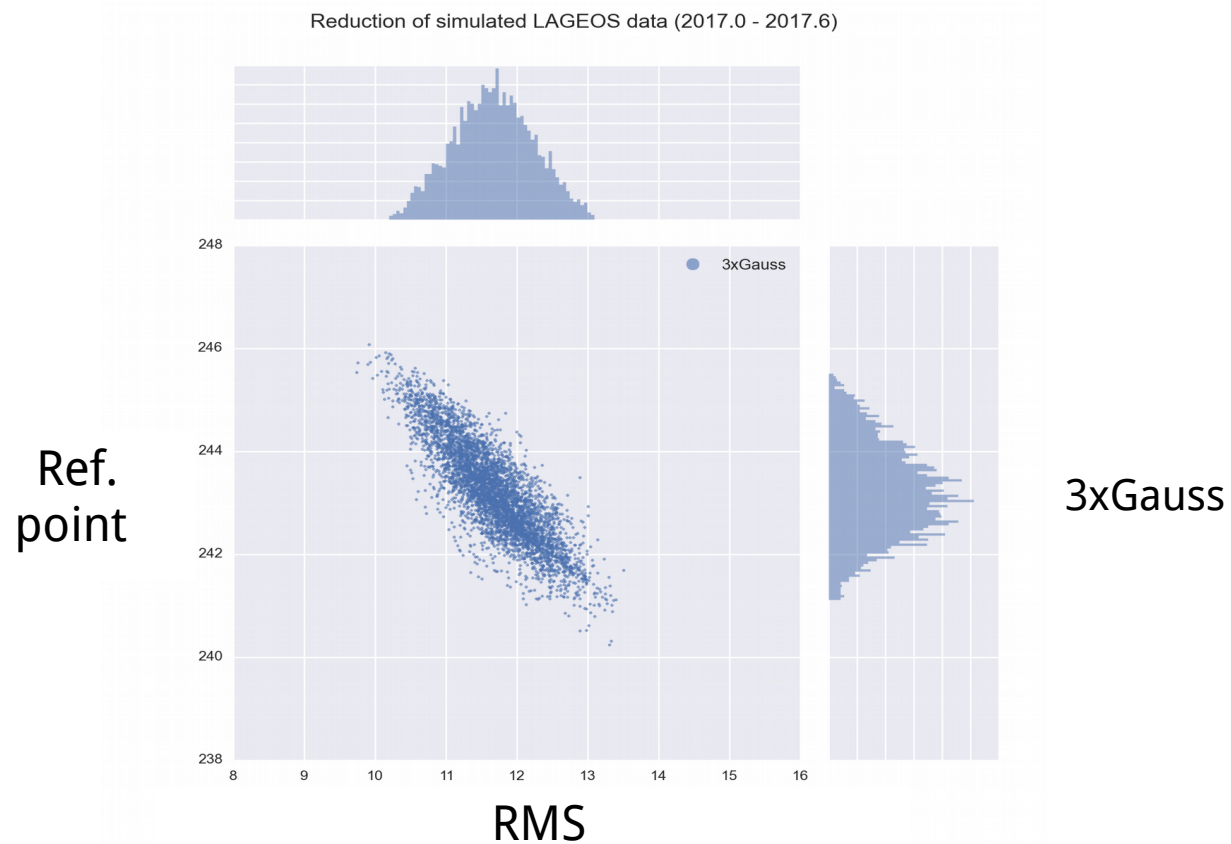Reduction of simulated LAGEOS data (2017.0 - 2017.6)

Ref. point

RMS
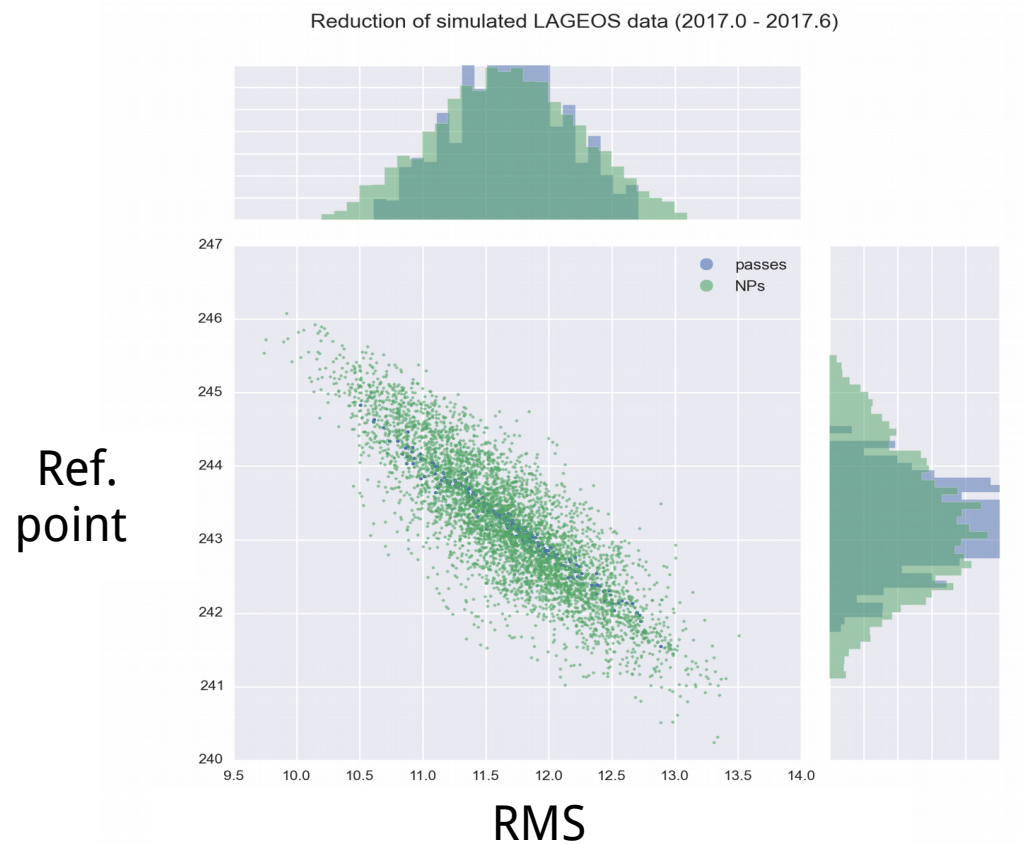
RMS peak-to-peak ~4 mm
Ref. Point peak-to-peak >5 mm

(Hx 3xGauss reduction method)

Individual NPs show a greater variability

NP reference point vs NP RMS

Reduction of simulated LAGEOS data (2017.0 - 2017.6)

Ref. point

RMS

3xGauss

We can compare different reduction strategies; e.g. Gauss vs iterative means

NP reference point vs NP RMS

Reduction of simulated LAGEOS data (2017.0 - 2017.6)

Ref
poir

3xGauss

2.5 x Iterative
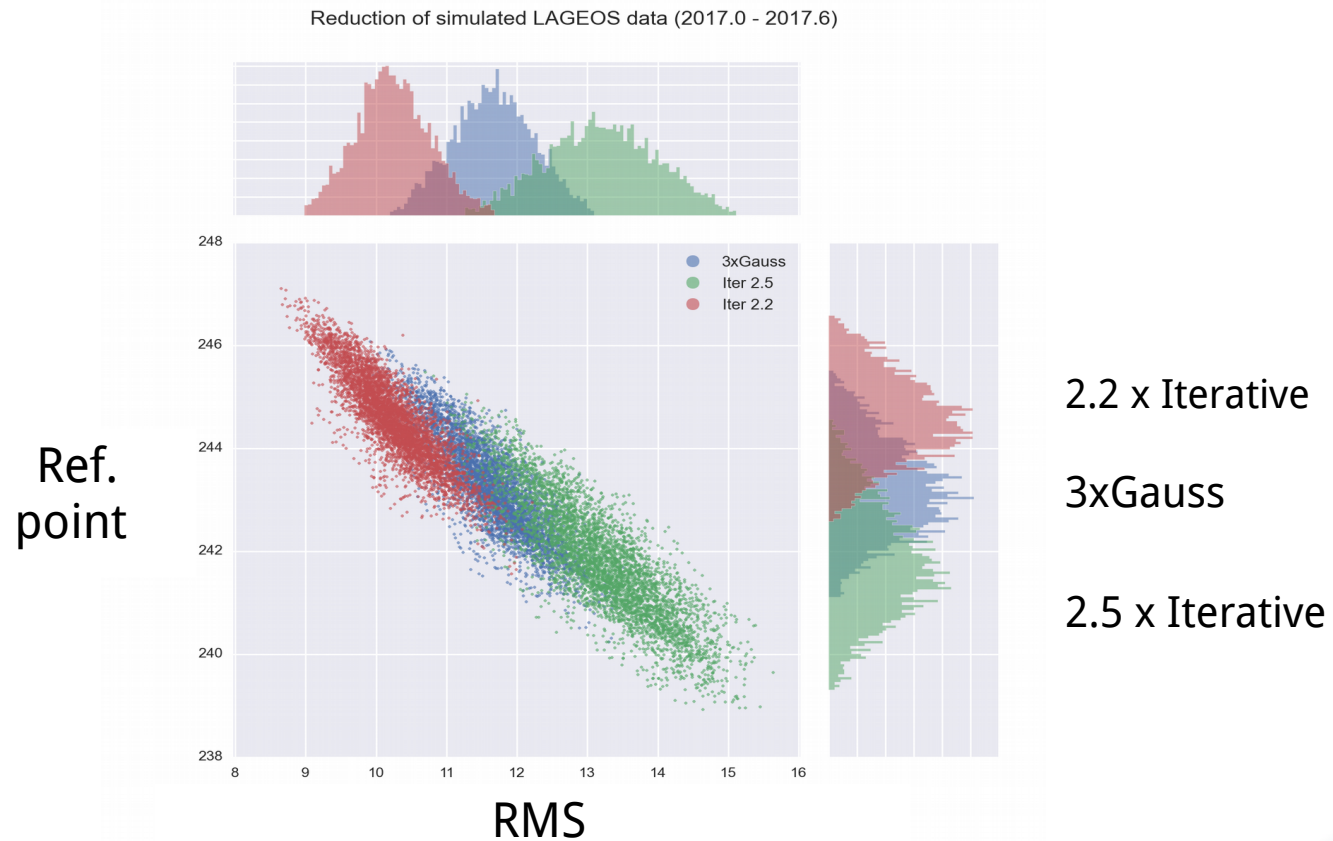
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NP reference point vs NP RMS

Reduction of simulated LAGEOS data (2017.0 - 2017.6)

Ref. point
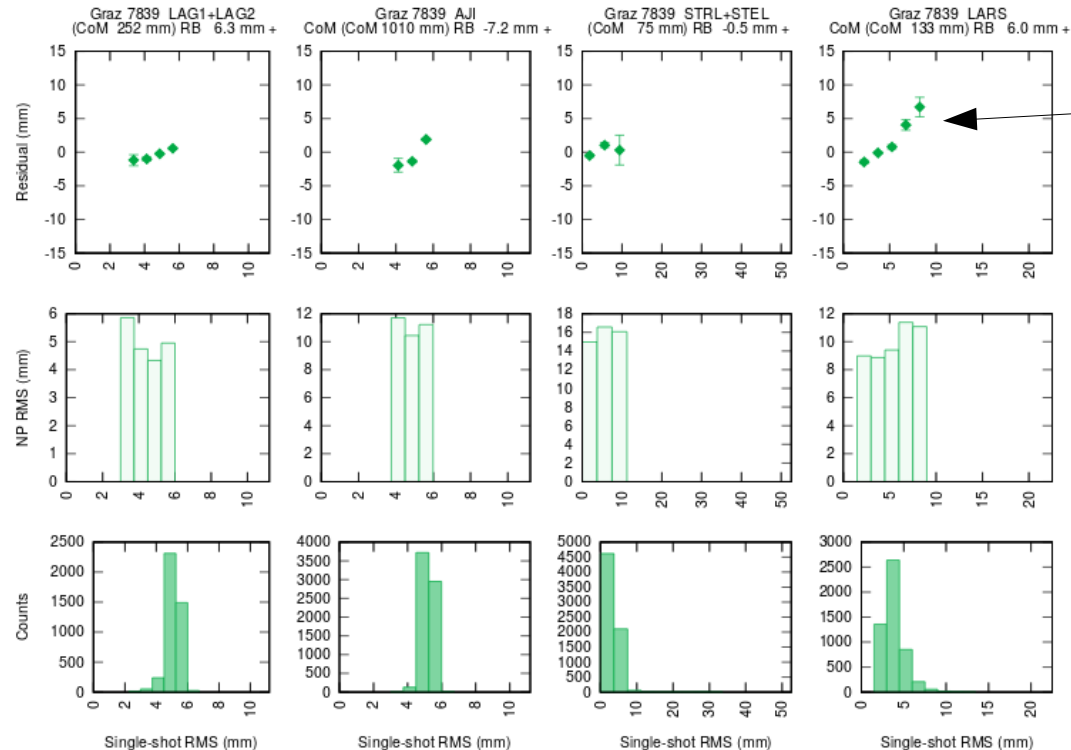
RMS

2.2 x Iterative

3xGauss
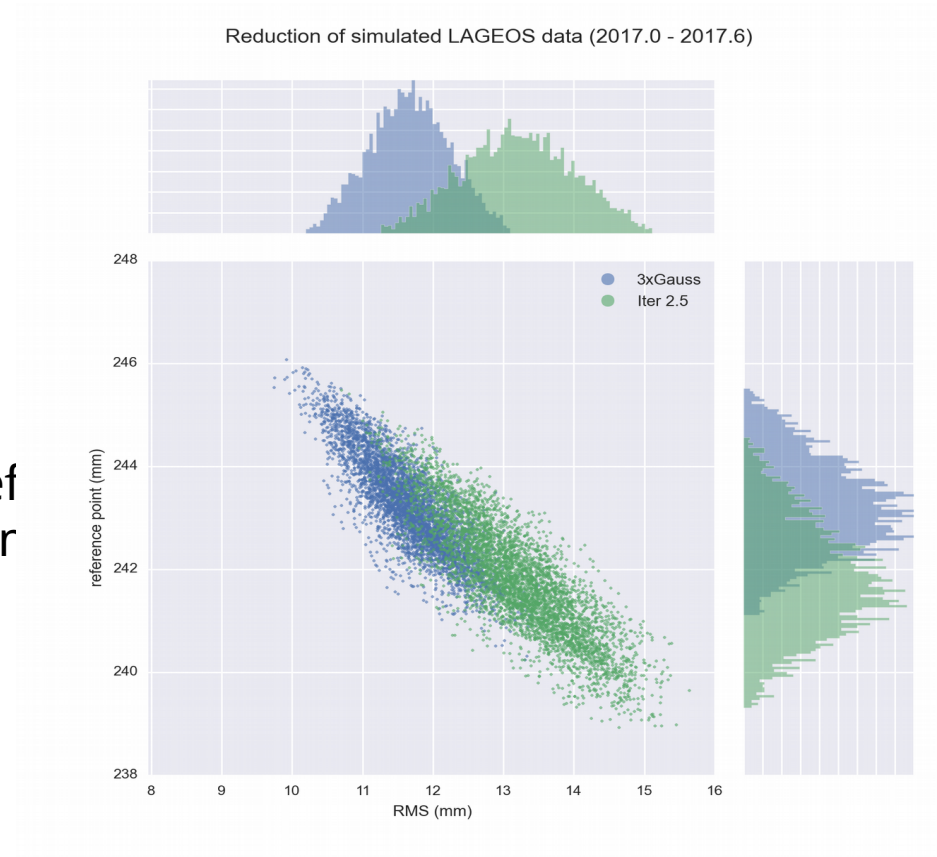
2.5 x Iterative

Similar performance, these methods are all very **sensitive** to changes in the **shape** of the distributions of detections

Leading edge methods?
Graz, a **very** similar station to Herstmonceux, does not show the Residual vs RMS correlation

Hitotsubashi Univ. analysis: NP residual vs NP σ (c5++ POD, 2016.5 – 2017.4)

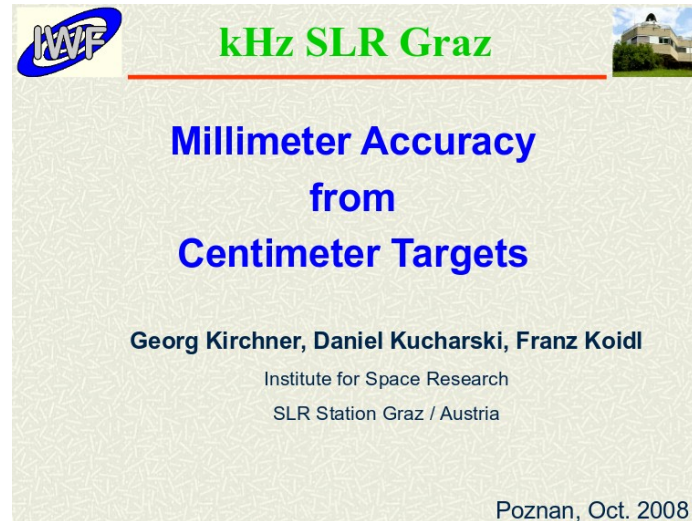slope for LARES
(no LE method is
used?)
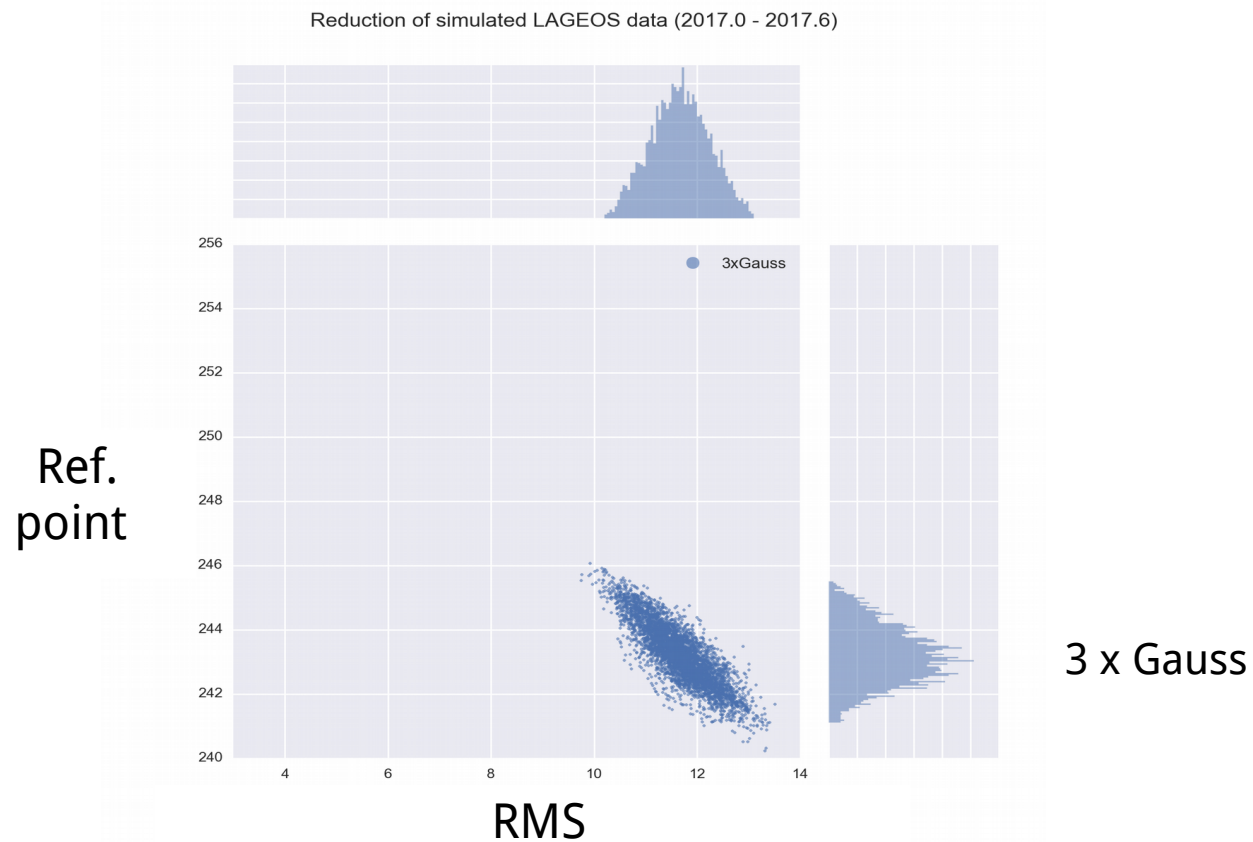
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Graz, a very similar station to Herstmonceux, does not show the Residual vs RMS correlation

LE (-a, +b) = average of data within (LEHM – a, LEHM + b) cm

Kicharski D., Kirchner G., Koidl F. A method to calculate zero-signature satellite laser ranging normal points for millimeter geodesy – a case study with Ajisai. EPS, 2015

NP reference point vs NP RMS

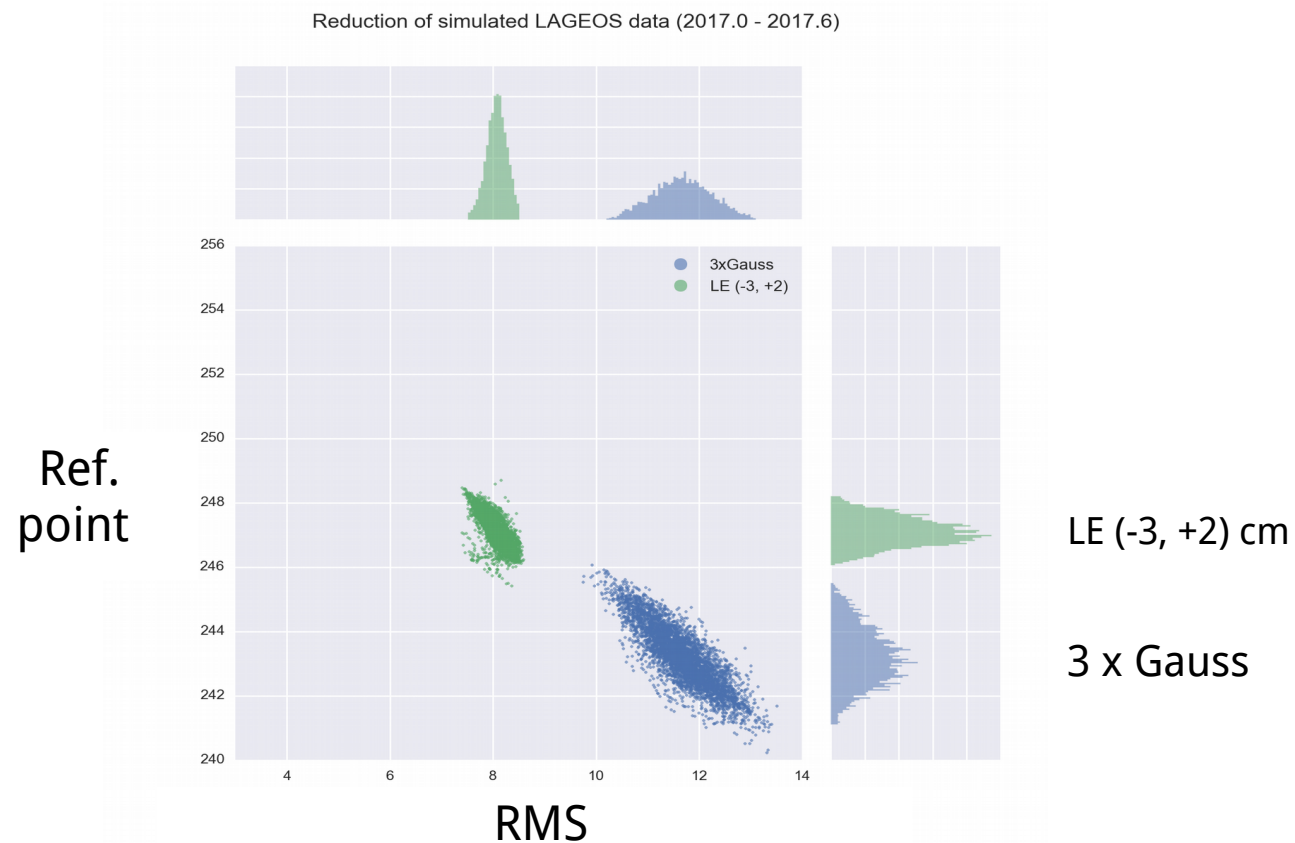Reduction of simulated LAGEOS data (2017.0 - 2017.6)

Ref. point

RMS

3 x Gauss

3 x Gauss vs leading edge methods
LE (-a, +b) = average of data within (LEHM – a, LEHM + b) cm

NP reference point vs NP RMS

Reduction of simulated LAGEOS data (2017.0 - 2017.6)

Ref. point

RMS

LE (-3, +2) cm

3 x Gauss

3 x Gauss vs LE methods
LE (-a, +b) = average of data within (LEHM – a, LEHM + b) cm

NP reference point vs NP RMS

Reduction of simulated LAGEOS data (2017.0 - 2017.6)

Ref. point

RMS

LE (-2, +2) cm

LE (-3, +2) cm

3 x Gauss

3 x Gauss vs LE methods
LE (-a, +b) = average of data within (LEHM – a, LEHM + b) cm

NP reference point vs NP RMS

Reduction of simulated LAGEOS data (2017.0 - 2017.6)

3 x Gauss vs LE methods
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Reduction of simulated LAGEOS data (2017.0 - 2017.6)
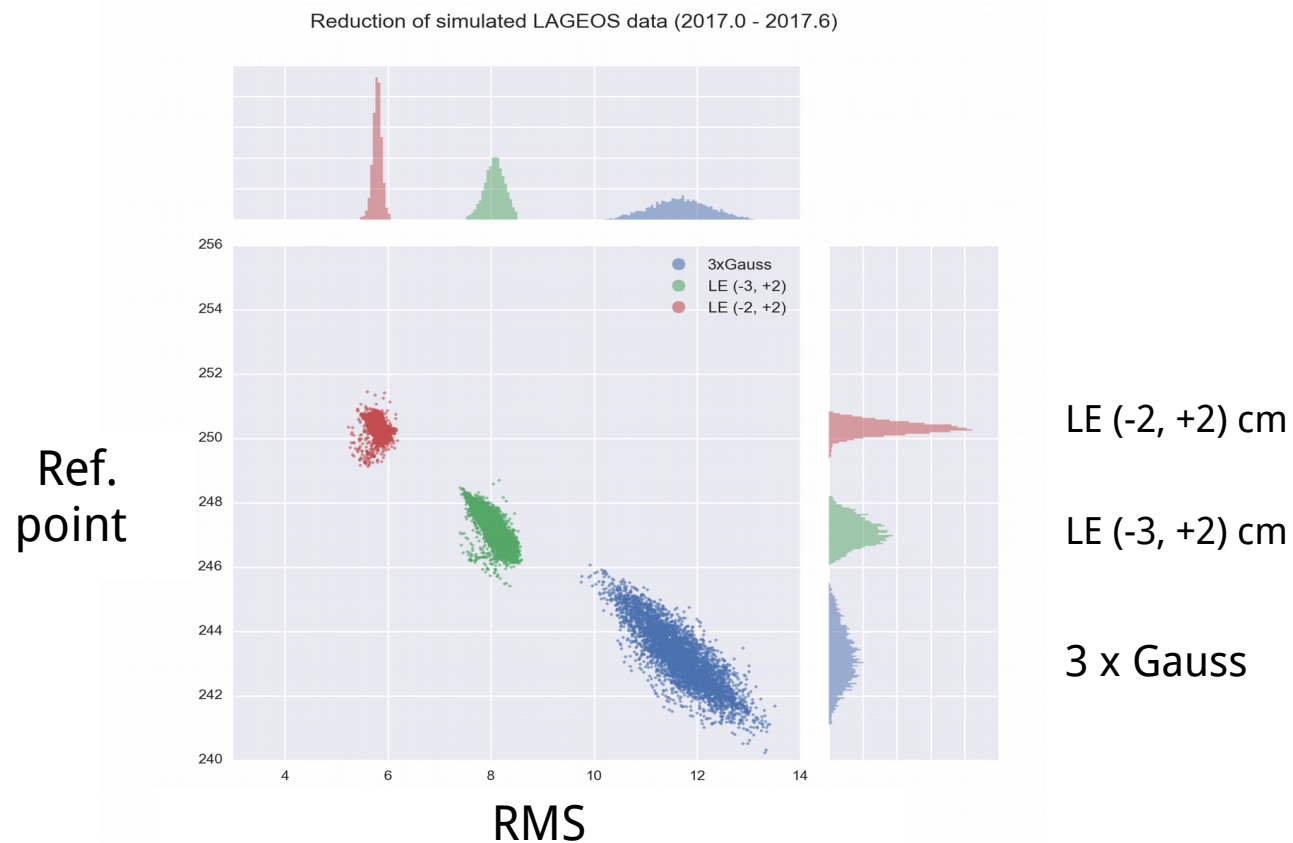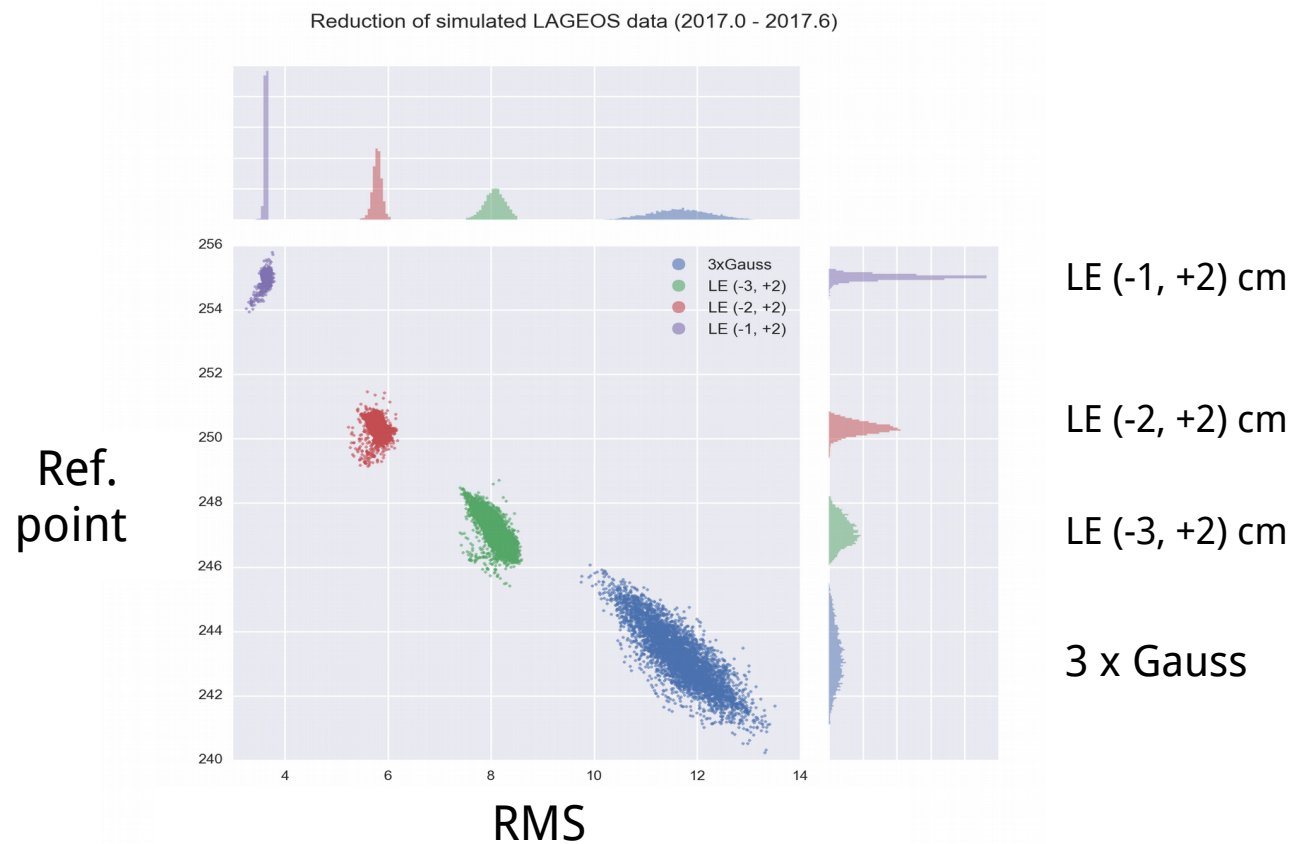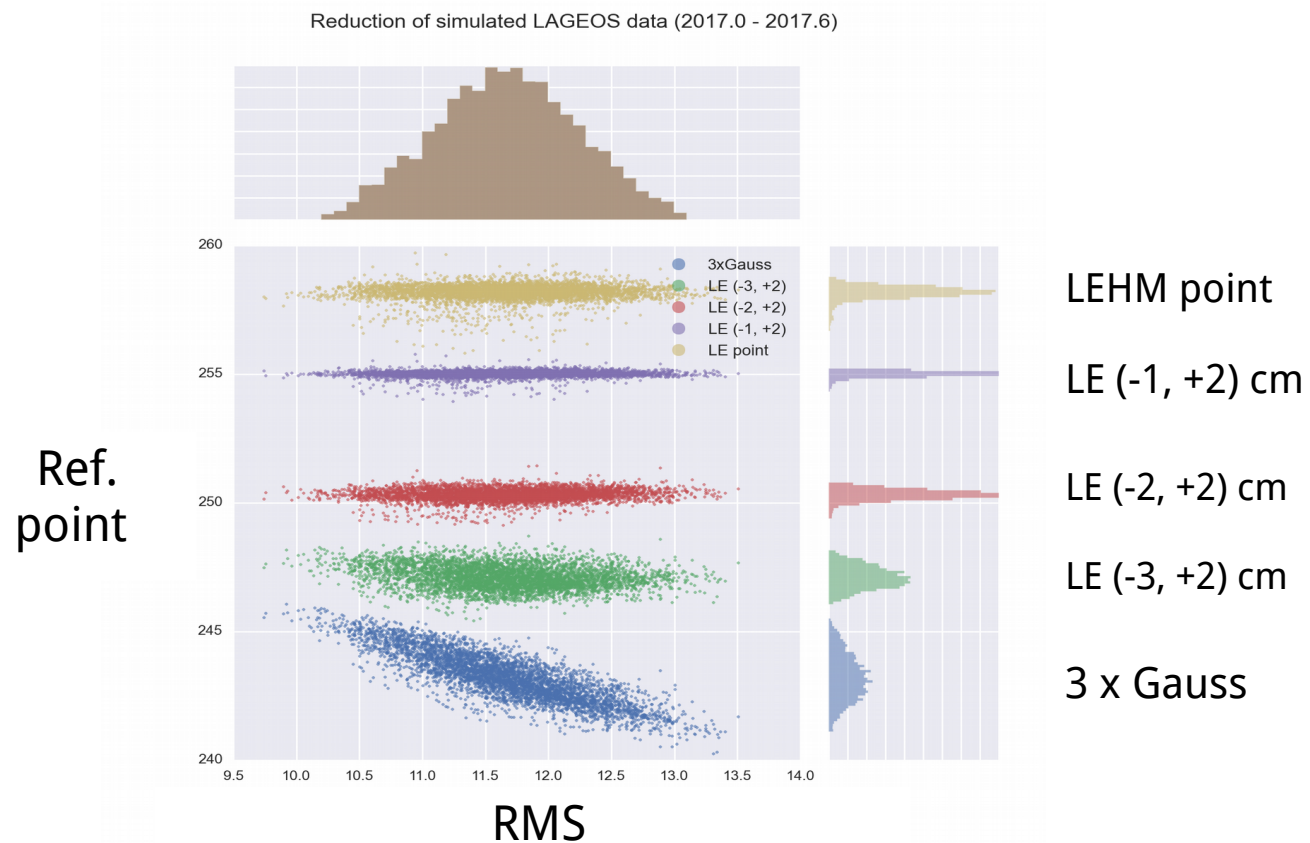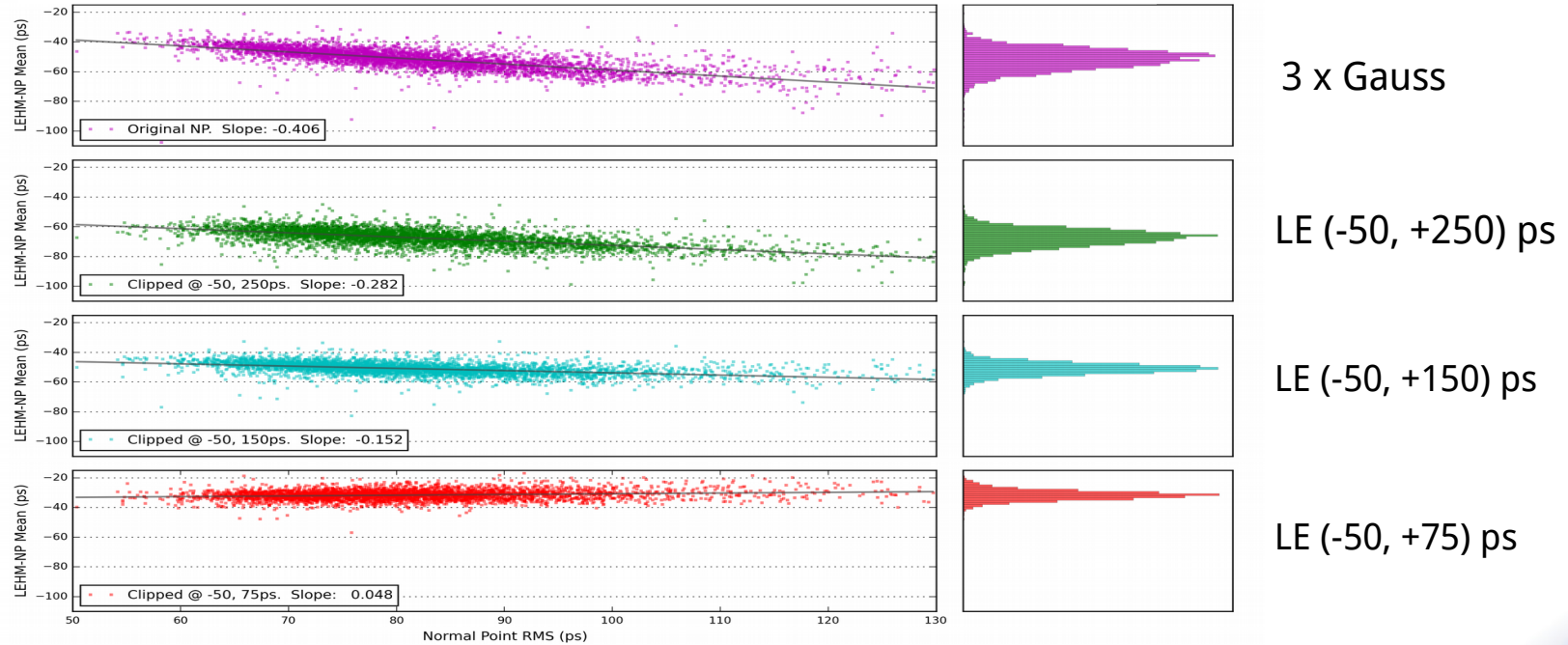
Fixed clipping from leading edge method much more **resilient** to underlying NP variability
LEHM point also stable (not the best), but tricky to determine accurately with real data

Testing with real data

LEHM - MEAN

3 x Gauss

LE (-50, +250) ps

LE (-50, +150) ps

LE (-50, +75) ps

M. Wilkinson: reduction of LAGEOS full-rate data using different clipping methods: optimal clipping level? Reliable LEHM determination from empirical distributions? Effect of individual cube corner signatures? Residual flattening methods?

Actual reflection point is unknown in real data, but **LEHM – MEAN** is an internal NP stability check

Averages of tighter leading edge clipping shown to perform better (LAGEOS, Ajisai and LARES tested)
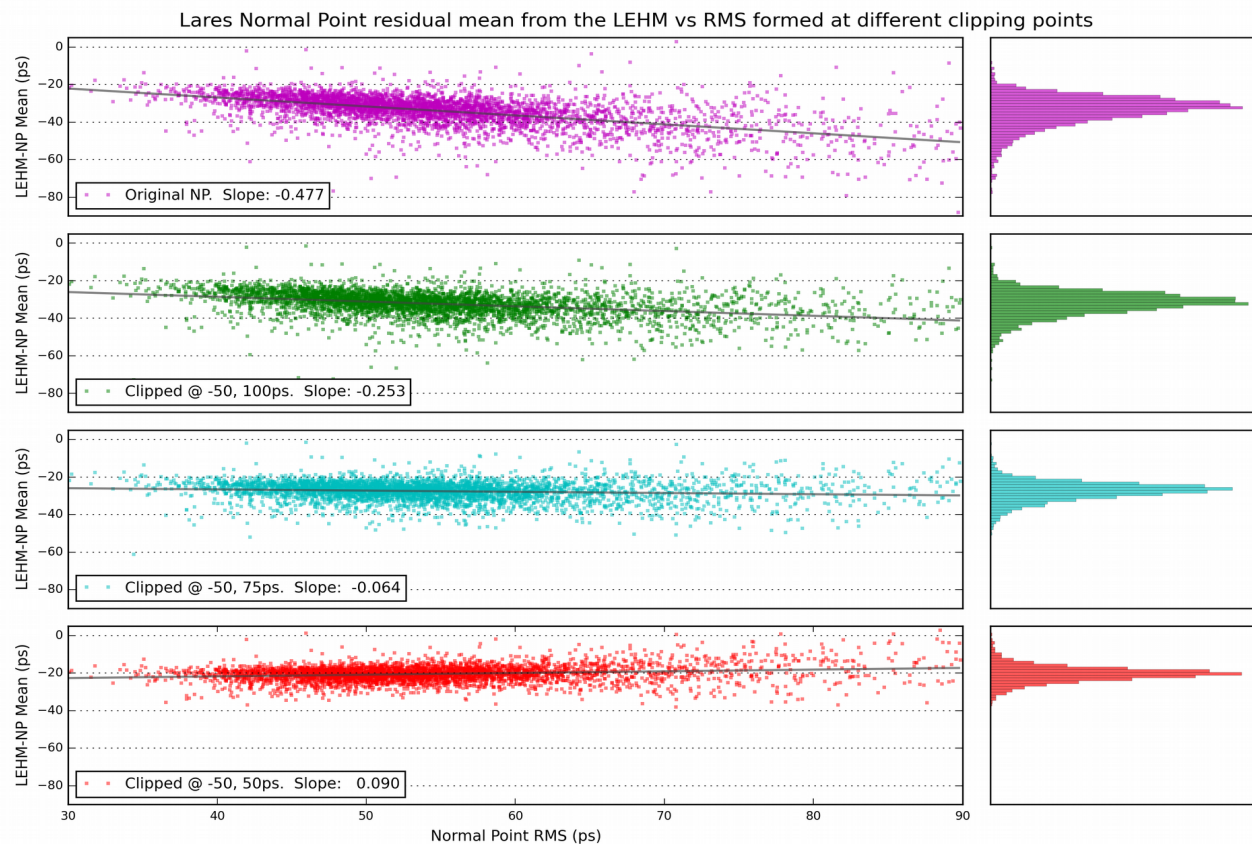
Summarising:

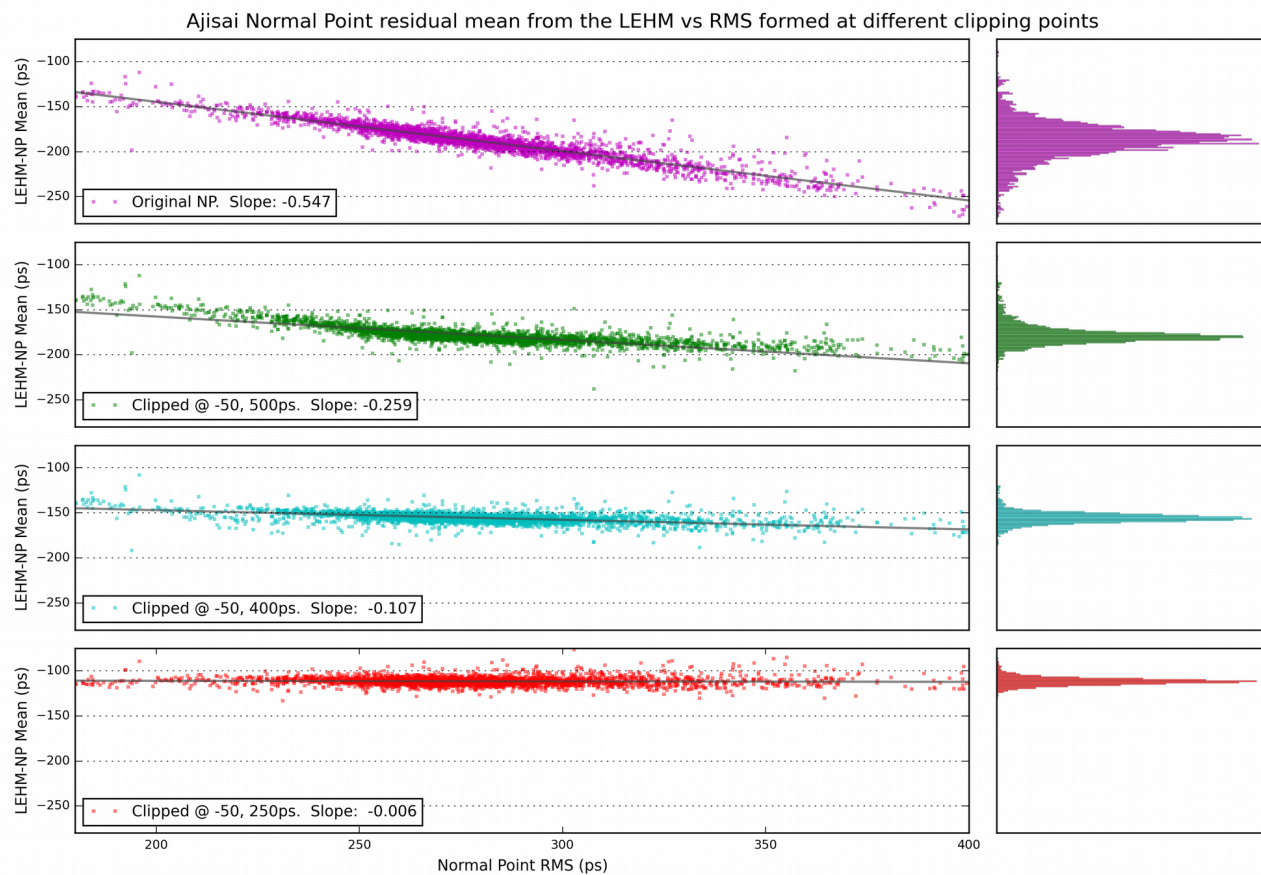Three effects that impact NP RMS have been considered

- **Limited physical sampling** of the LRA induces variability in the NPs reference points that explains a good part of what we see in the orbital analysis residuals

- **Statistical sampling variability** causes additional spread to both NP reference points and RMS, but does not greatly contribute to their correlation

- **Background noise** during daytime at very low return rate may stretch the distribution of NP RMS towards higher values (and lower reference points)

Other:

- LE-based reduction methods perform very well

- The practicalities involved in implementing alternative reduction methods, and their sources of error, have not been considered

- Different clipping methods require, of course, appropriate **centre of mass** values

Thank you

Lares Normal Point residual mean from the LEHM vs RMS formed at different clipping points

Original NP. Slope: -0.477

Clipped @ -50, 100ps. Slope: -0.253

Clipped @ -50, 75ps. Slope: -0.064

Clipped @ -50, 50ps. Slope: 0.090

Normal Point RMS (ps)

LEHM-NP Mean (ps)

Ajisai Normal Point residual mean from the LEHM vs RMS formed at different clipping points

Original NP. Slope: -0.547

Clipped @ -50, 500ps. Slope: -0.259

Clipped @ -50, 400ps. Slope: -0.107

Clipped @ -50, 250ps. Slope: -0.006

NP mean and RMS vs Pass mean and RMS

NP means and RMS values are correlated with those of the passes they belong to, as expected. However, the intra-pass variability is quite big. In the case of the means, in the vast majority of passes the range of NP means include the overall reference point mean

Pass RMS is therefore a poor predictor of NP means. Also, for both pass and NP data, we are seeing deviations from their average expected values, rather than biased estimates → not systematic error